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Expanding the Spanish high-speed railway network

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ABSTRACT

This paper presents a model for the expansion of transportation networks incorporating specific requirements about population coverage, budget constraints, intermediate goals and origin–destination flows, among others. The model is applicable to the current expansion project of the Spanish high-speed railway network that has been proposed by the Spanish Government under the program Strategic Planning of Infrastructure and Transport (see source [c]).

Our approach looks for solutions that may be used as additional information in the decision-making process of any network expansion. We report on the application of this methodology to the Spanish railway network and on a computational experience based on simulated data varying the number of cities and time horizons which proves the efficiency of the proposed algorithm.

1. Introduction

The Strategic Planning of Infrastructure and Transport (PEIT, see source [c]), approved by the Council of Ministers of the Spanish Government on July 15, 2005, defines an execution plan in Spanish infrastructure and transport for the period 2005–2020. This plan sets the general criteria to guide all the decisions to be made in this area in Spain in the years to come.

With PEIT, the Spanish Ministry of Public Works assumes publicly available binding compromises as a tool to develop the expansion project of the Spanish railway network within a given planning horizon and with known characteristics. Apart from the specific goals directly related to the new network, with PEIT the Government also looks for some additional goals: economic development, competitiveness, social and territorial cohesion, and quality of life of citizens. These goals are pursued by means of a set of measures leading to achieve a more integrated, safe, efficient and environmentally respectful transportation system.

PEIT realization will imply the largest investment in infrastructure in Spain up to date, forecasting a total budget of 248 892 millions €(m€), i.e., a year average investment of more than 15 000 m€. This figure is around 1.5% of the Spanish GDP (gross domestic product) during the time horizon of PEIT. The main element of PEIT is its action on the railway network, including an ambitious high-speed railway network expansion, that will cover all the country. The forecasted investment for this action is around 50% of the entire budget, whereas more than 67.6% of the former is devoted to the expansion of the high-speed railway network and mixed traffic.

The Spanish high-speed railway network (tracks that allow speed faster than 250 km/h) had at the beginning of PEIT (2005), an overall length of 1031 km, crossing the Iberian Peninsula diagonally SW-NE (Sevilla-Madrid-Zaragoza-Lleida/Huesca). The expansion of the network tries to avoid the concentration and centralization of the economic activity in a number of big centers weakening the areas of minor potential in benefit of the dominant ones. Its goals are: (1) to expand the current network, starting with the current axis, so that the Spanish high speed network joins the international frame, and (2) to construct an interprovincial transportation system of quality.

According to the established guidelines, the legal regulations and the public promises of politicians in charge of the Department of Public Works in 2005, the final picture of the railway network for the year 2020, after the application of PEIT, will be the following one:

1. The total length of the Spanish railway network will be almost 10 times the one at the beginning: from 1031 to 10 000 km. This fact will imply to build around 9000 km in 15 years, whereas only 1031 km have been built in the last 15 years.
2. At least 90% of the population must be within a radius of 50 km from a station of the high-speed railway network, and there must be a station in every "major city" in Spain.

Therefore, as a consequence of PEIT, it will be necessary to design a plan of expansion for the next 15 years fulfilling the

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above-mentioned objectives and making a responsible use of public resources.

The goal of this paper is to develop a tool for supporting the decision making process of the expansion of the Spanish transportation network according to the goals specified by PEIT. Our approach provides a tool for helping the decision makers on the expansion of transportation networks, that incorporate specific requirements about population coverage, O–D flows, budget constraints, intermediate goals, etcetera. It is clear that different requirements on the patterns of temporal inter-city connections, connectivity in any stage of the planning horizon or required amount of origin–destination flows per periods will imply different sequences of intermediate achievements. Moreover, it is important to highlight that the consideration of "economic", "social" and "environmental" goals may lead to simultaneously consider several objectives. The level of achievement and the hierarchy that the decision-maker establishes among the different goals will determine the model to be considered and, therefore the result for the network expansion.

The case study that motivated our analysis, i.e. the Spanish PEIT, establishes neat social requirements as goals to be achieved (population coverage, maximal distance to the closest station …) whereas nothing is clearly stated in terms of environmental impact of the final design. Therefore, in our analysis we have chosen an approach that looks for solutions that ensure all requested goals at a minimum overall construction (due to opening links and nodes) plus operation (due to O–D flows) costs. Needless to say that although we give the highest priority to cost, all the remaining requirements appear as constraint satisfaction levels that must be fulfilled by any solution. Alternative approaches could have been implemented. Among them we had considered using a multiobjective model that looks for non-dominated solutions with respect to both economic (construction plus operation) and social costs; and the maximization of net benefits. The former was very appealing, although it led to some problems in estimating economically social welfare and also in the difficulty of the problem to be solved. The latter, i.e maximization of net benefits, seemed not to be appropriate in this case since this type of public investment does not look for benefits in the mid or long term but for creating infrastructure that promote the economy and social cohesion. (Hence, hard to be measured in terms of net gains.)

With this tool we aim to develop quantitative mechanisms for evaluating alternatives in the design of transportation networks. These mechanisms are given by mathematical programming tools that allow to incorporate different goals by means of new constraints that model the actual situation. Each alternative design of the expansion of a network is motivated by the application of different criteria as well as by the analysis done on the different scenarios. For further details on decision support systems, see the books by Sprague and Watson [36] and Turban [38].

As indicated above, we will apply our methodology to an actual case according to the goals given by the Spanish PEIT. Methodologically, in our case, this fixes the parameters in the mathematical programming model that will evaluate the different alternatives of network expansion. Some related references addressing the use of operations research techniques to support public decisions are [4,12,22,26,41].

Our problem is closely related with a category of combinatorial optimization models known as "design of networks". This connection is very useful when we model our framework space as a graph and it has been successfully exploited in several areas, provided that one wants to establish an "ideal design" usually chosen among some pre-specified types. Usually, these problems appear in areas such as power distribution, communication, and computer and transportation networks, among others. There is a large number of references that approach these problems from an optimization point of view. The interested reader is referred to Magnanti and Wong [27], Goemans and Williamson [19], Raghavan and Magnanti [33], Balakrishnan et al. [2], Berbeglia et al. [5], Hinojosa et al. [21], Luss and Wong [25], Pióro and Medhi [30] and the book edited by Magnanti et al. [3] for many other applications.

The first one, [27], approaches many aspects of network optimization problems in general, and, especially, those related to the design of networks. The recent texts by Pióro and Medhi [30] and Costa et al. [9] approach also different aspects of the design of a network. From a general perspective, we can mention, for example, the chapter on optimal trees by Magnanti and Wolsey in [3, Chapter 9]. In addition, we can mention the works by Gouveia [18], Puerto and Tamir [31] and Tamir et al. [37], that study spanning trees from a perspective of location that can be seen as design of networks. We also cite some classic references dealing with the design of transportation networks. Vuchic [40] recommends some rules for the design of railroad networks, Rothengatter [32] does the same for road networks, and Scott [35] for an integrated case. Chung and Oh [8] and Hong et al. [23] give different approaches for train-set routing and train sequencing, respectively. The interested reader is referred to the more recent references by Bielli et al. [6], Button [7], Crainic [11], Drezner and Wesolowsky [13], Goossens's [15], Grötschel et al. [16], Holder [20] and the references therein for further details on the design of networks.

In spite of the fact that at times simulation has been used as a methodological tool for approaching these problems (see the works by Siefer and Böcker [39]), the most successful approaches to deal with these problems use optimization models borrowed from mathematical programming. In this line, we mention the seminal works by Asad [1] and Crainic, Ferland and Rousseau [10], as well as the most recent by Guglielminetti et al. [17] or Peterson and Taylor [29], related to the railway network in Switzerland and Brazil, respectively. The reader may note that those references refer to the design of operation plans, rather than to that of the design of the network. This means a significant difference with regard to the problem that we deal with.

In general, networks design problems are computationally cumbersome and in most cases intractable (see [3]). Apart from exact methods for solving integer programming problems, one can use heuristic methods for obtaining approximated solutions faster than solving the exact problems. In [14,34,24] the reader can find some heuristic approaches for obtaining solutions of integer programs, in general, with some applications to problems of network design. We prove that our problem is NP-hard and show, by our computational experiments, that exact resolution times increase very quickly. Therefore, we develop a scatter search heuristic algorithm (see [34] for further details) that provides good solutions for these types of problems with much less computational effort.

The paper is organized as follows. The first section is our introduction. Here, we describe the problem, the case study, review specialized literature and state our goals. The second section introduces the general model defining the set of variables and constraints. Section 3 develops a heuristic scatter-search algorithm that provides upper bounds for the optimal solution reducing the running times of general purpose solvers as CPLEX or XPRESS. This algorithm is based on combining the dynamic generation of partial solutions at each period, with an improving phase which includes a randomization component. Here, we also present our computational tests run on a battery of randomly generated problems and compare the results obtained both by the heuristic algorithm and the optimal solution given by the commercial solver XPRESS. Section 4 applies our methodology to the case of the Spanish high-speed railway network showing that alternative solutions to the one implemented by the Spanish Government are possible still fulfilling all the requirements stated in the Strategic Planning of Infrastructure and Transport (PEIT). This section also compares the solutions obtained by the exact and heuristic methods for the expansion of the Spanish railway network validating the usage of the heuristic when applied to real data. Finally, Section 5 draws some conclusions on the paper.

2. General model

In this section, we present a general setting (a mathematical programming model) for addressing the expansion of existing railway networks, based on specific quantitative requirements (population coverage, length, budget, O–D flows,…).

In order to present the general model we introduce the following notation: let M be the number of cities (numbered from 1 to M) and let T be the planning horizon. We assume that the configuration at the initial period ($t=0$) is known in advance. In addition, we denote by

$\mathcal{M}=\{1,\ldots,M\}$ and $\mathcal{T}=\{0,\ldots,T\}$

The model is defined by a set of parameters that we list below:

- Unit building cost of an edge in every period: $\{c^t\}_{t\in\mathcal{T}}$.
- Distance matrix between pairs of nodes: $\{d_{ij}\}_{i,j\in\mathcal{M}}$.
- Cost to open a node (to build a station in a city) in every period: $\{e_i^t\}_{i\in\mathcal{M}}^{t\in\mathcal{T}}$.
- Demand per node (population of each city) at each period: $\{m_i^t\}_{i\in\mathcal{M}}^{t\in\mathcal{T}}$.
- Lower bound on the proportion of population to be covered by the network: $\mathtt{pTOT}$.
- Total budget: $\mathtt{B}$.
- Upper bound on the maximal edge length: $\mathtt{dMAX}$.
- Lower bound on the minimal edge length: $\mathtt{dMIN}$.
- Lower bound on the total network length: $\mathtt{dTOT}$.
- Minimum proportion of budget spent at period t: $\delta^t, t\in\mathcal{T}$.
- Flow (number of passengers) sent from origin i to destination j in the long run: $w_{ij}, i,j\in\mathcal{M}$.
- Overall flow originated from origin i: $O_i=\sum_{j=1}^M w_{ij},\ i\in\mathcal{M}$.
- Overall flow with destination j: $D_j=\sum_{i=1}^M w_{ij},\ j\in\mathcal{M}$.

The decision variables of the model are

$$y_j^t=\begin{cases}1 & \text{if a new station is opened at city } j \text{ in period } t\\ 0 & \text{otherwise}\end{cases}$$

$$x_{ij}^t=\begin{cases}1 & \text{if edge } (i,j) \text{ is built in period } t\\ 0 & \text{otherwise}\end{cases}$$

Note that we consider only variables x_{ij}^t with $i<j$ to avoid duplication since links between nodes are undirected.

p^t = Percentage of population to be covered in period t,
z^t = Proportion of the budget to be spent in period t,
f_{ij} = Flow starting from origin i, initially routed via j,
v_{ilj} = Flow starting from origin i with final destination j initially routed via l,
u_{ilj} = Flow starting from origin i with final destination j and finally routed via l.

The goal is to minimize the overall cost of expansion of the network plus the operation costs due to the flow of passengers in the long run. Thus using the above notation the objective function has the following expression:

$$\min \sum_{t=1}^{T}\sum_{i=1}^{M-1}\sum_{j=i+1}^{M} c^t d_{ij}x_{ij}^t+\sum_{t=1}^{T}\sum_{i=1}^{T} y_i^t e_i^t + \sum_{i=1}^{M}\left(\sum_{k=1}^{M}\alpha d_{ik}f_{ik}+\sum_{j=1}^{M}\sum_{l=1}^{M}\beta d_{ij}v_{ilj}+\sum_{l=1}^{M}\sum_{j=1}^{M}\varepsilon d_{ij}u_{ilj}\right)$$

The objective function accounts for the minimization of the overall construction cost over the entire planning horizon plus the long run flow cost of the network, i.e. when the planning horizon has finished and the final design is settled. The first part of the objective gives the cost of the different links whereas the second one defines the cost for opening new nodes (stations). The third term accounts for the operation cost (flow cost), α,β and γ are the economy of scale factors; unit cost of long distance connections that need to be routed via intermediate nodes (stations) must be smaller than short, direct connections, so $0<\beta,\ \gamma\le\alpha$.

As for the constraints defining the model we have the following.

Each edge and each node is built at most once in the entire planning horizon:

$$\sum_{s=0}^{T} x_{ij}^s\le 1,\quad i\in\mathcal{M},\ j>i \tag{1}$$

$$\sum_{s=0}^{T} y_i^s\le 1,\quad i\in\mathcal{M} \tag{2}$$

An edge is built only if its origin and destination nodes are built:

$$x_{ij}^t\le\sum_{s=0}^{t} y_i^s,\quad x_{ij}^t\le\sum_{s=0}^{t} y_j^s\quad i\in\mathcal{M},\ t\in\mathcal{T},\ j>i \tag{3}$$

Let us denote by $\mathtt{P}^t=\sum_{i=1}^M m_i^t$ the estimated population at period $t\in\mathcal{T}$, whereas p^t and z^t are the corresponding percentages of population and budget covered at period $t\in\mathcal{T}$. Thus, we have

$$\sum_{i=1}^{M} y_i^t m_i^t\ge p^t\mathtt{P}^t,\quad t\in\mathcal{T}\backslash\{0\} \tag{4}$$

$$\mathtt{pTOT}\le\sum_{t=0}^{T} p^t\le 1 \tag{5}$$

$$\sum_{i=1}^{M-1}\sum_{j=i+1}^{M} c^t\cdot d_{ij}\cdot x_{ij}^t+\sum_{i=1}^{M} y_i^t\cdot e_i^t\le z^t\cdot\mathtt{B},\quad t\in\mathcal{T}\backslash\{0\} \tag{6}$$

$$\sum_{t=1}^{T} z^t\le 1,\quad z^t\ge\delta^t\quad t\in\mathcal{T}\backslash\{0\} \tag{7}$$

A node is opened if the edge connecting it with some other node is built in a previous, the same, or the next period:

$$y_i^t\le\sum_{s=0}^{\min\{t+1,T\}}\sum_{i<j}x_{ij}^s+\sum_{s=0}^{\min\{t+1,T\}}\sum_{i>j}x_{ji}^s,\quad i\in\mathcal{M},\ t\in\mathcal{T}\backslash\{0\} \tag{8}$$

The overall length of the expanded network is bounded from below by $\mathtt{dTOT}$:

$$\sum_{t=0}^{T}\sum_{i=1}^{M-1}\sum_{j=i+1}^{M} d_{ij}x_{ij}^t\ge \mathtt{dTOT} \tag{9}$$

Edges of length longer than $\mathtt{dMAX}$ or shorter than $\mathtt{dMIN}$ are not allowed:

$$(\mathtt{dMAX}-d_{ij})\cdot x_{ij}^t\ge 0,\quad (\mathtt{dMIN}-d_{ij})\cdot x_{ij}^t\le 0,\quad i,\ j\in\mathcal{M},\ i<j,\ t\in\mathcal{T}\backslash\{0\} \tag{10}$$

Variables concerning the initial configuration (period 0) are fixed. Thus, if S_x^0, S_y^0 denote, respectively, the existing links and nodes at period 0, we have

$$y_i^0 = 1, \quad i \in S_y^0 \subseteq \mathcal{M} \tag{11}$$

$$x_{ij}^0 = 1, \quad (i,j) \in S_x^0 \subseteq \mathcal{M} \times \mathcal{M} \tag{12}$$

At the end of the planning horizon each "important" node must be open (settled as a station):

$$\sum_{t=0}^{T} y_i^t = 1, \quad i \in \mathcal{M} \tag{13}$$

The final network must be connected. This is ensured imposing, when necessary, that no subset $S \subset \mathcal{M}$ of nodes is disconnected from its complement $\mathcal{M} \backslash S$:

$$\sum_{t=0}^{T} \sum_{i \in S} \sum_{j \in \mathcal{M} \backslash S} x_{ij}^t \geq 1, \quad \forall S \subset \mathcal{M} \tag{14}$$

As for the operational constraints we have the following. The long run flow originated at node $i \in \mathcal{M}$ must be equal to O_i:

$$\sum_{j=1}^{M} f_{ij} = O_i, \quad i \in \mathcal{M} \tag{15}$$

The O–D flow from nodes i to j, $i,j \in \mathcal{M}$ must be equal to w_{ij} regardless of the way it is routed via intermediate nodes:

$$\sum_{l=1}^{M} u_{ilj} = w_{ij}, \quad i,j \in \mathcal{M} \tag{16}$$

$$\sum_{l=1}^{M} v_{ilj} = w_{ij}, \quad i,j \in \mathcal{M} \tag{17}$$

Flow originated in a node, say i, can be firstly routed via l only if link (i,l) has been built at any period:

$$\sum_{j=1}^{M} u_{ilj} \leq O_i \sum_{t=1}^{T} x_{li}^t, \quad i,l \in \mathcal{M} \tag{18}$$

Flow originated in a node, say i, can be lastly routed via l only if link (l,j) has been built at any period:

$$\sum_{i=1}^{M} v_{ilj} \leq D_j \sum_{t=1}^{T} x_{jl}^t, \quad l,j \in \mathcal{M} \tag{19}$$

Flow between nodes is conserved:

$$\sum_{l=1}^{M} v_{ikl} + \sum_{j=1}^{M} u_{ikj} - \sum_{l=1}^{M} v_{ilk} - f_{ik} = 0, \quad i,k \in \mathcal{M} \tag{20}$$

Finally, the ranges for the variables in the model are the following:

$$v_{ikl}, u_{ikj}, f_{ik} \geq 0, \quad i,k,j \in \mathcal{M} \tag{21}$$

$$x_{ij}^t, y_j^t \in \{0,1\}, \quad i,j \in \mathcal{M} (i<j), \; t \in \mathcal{T} \tag{22}$$

We observe that even the subproblem on the x variables, i.e. removing the flow variables u, v, f and the location variables y, contains knapsack as a subproblem because of constraint (9). Therefore, the original problem is NP-hard. Moreover, according to our computational experience, this problem is very hard to solve to optimality. As an alternative, in the next section, we propose a method that reduces the execution times of general purpose solvers to obtain the exact solution, by means of a heuristic algorithm.

3. A heuristic approach

In the application of the general model to the case study of the Spanish high-speed railway network, general purpose solvers (XPRESS) were able to solve the problem up to optimality (see Section 4 for further details about the solution obtained for the Spanish case). Nevertheless, the NP-hardness of this type of problems lead us to propose some alternative solution procedures that may handle larger instances. To this end, we have developed a scatter search heuristic adapted to find good solutions for these problems. This section is devoted to present the details of our algorithm whereas some computational results on randomly generated data will be presented in Section 3.3.

Scatter search is an evolutionary method that has been successfully applied to a wide list of hard optimization problems. Scatter search constructs new trial solutions by combining a previously obtained solutions and employing strategic designs that exploit context knowledge. In contrast to other evolutionary methods like genetic algorithms, scatter search is founded on the premise that systematic designs and methods for creating new solutions afford significant benefits beyond those mainly derived from recourse to randomization. In our implementation we construct partial solutions up to a given stage based on four types of interchange operators. Our goal is to find solutions that locally minimize partial construction plus operation costs until the current stage. The method that we propose combines two phases: (1) obtaining a feasible partial solution for each period, and (2) a heuristic improvement of this solution based on our scatter search strategy.

Our algorithm consists of two basic components: *a construction phase* that produces a feasible solution at the beginning of each stage, and an adaptive search strategy with a probabilistic selection procedure to improve incumbent solutions (*improving phase*). These components are linked, resulting in an iterative method that, at each iteration constructs a feasible partial solution, and then at the final period, a locally optimal solution. The pseudocode presented in Algorithm 1 describes the entire algorithm. The different subroutines are later explained in the text.

Algorithm 1. Complete Procedure.

Input: An initial configuration of the network at period 0.
for *Each period* **do**
  Compute a partial solution for expanding the network in that period from the configuration of the previous period : `partialsolution`.
  for *each pair of cities* **do**
    Choose `type` in {1,2,3,4} and perform changes of edges given by `type` in order to evaluate the incumbent partial solution : `improvement(type)`.
Output: Heuristic Solution for expanding the network.

3.1. Construction phase

The construction phase of the algorithm builds a partial solution that is feasible up to the current period. This solution is improved by a local search in its neighborhood thus producing a locally optimal solution. In order to do that, for each i and j let $\sigma(i) = j$ denote that the city j is the ith element in the sorted list of entries with respect to the non-increasing values of O_i, the overall flow originated from city i.

Algorithm 2 shows the pseudocode for this phase. In that pseudocode, we denote by *cost*(t) and *flow*(t), respectively, the accumulated construction and network flow costs up to period t and by $B(t)$ a upper bound for the budget at period t.

Algorithm 2. `partialsolution`(t).

Input: An initial configuration for the network up to period $t-1$.
Sort the non-open nodes in $\mathcal{M}$ by non-increasing values of $O_i : \sigma(1), \ldots, \sigma(l)$.
while $cost(t) < B(t)$ **do**
  –Open node i in the ordering given by σ.
  –Link (by an edge) node i with the closest open node.
  –Update $cost(t)$ and $flow(t)$.
Output: A feasible solution for stage t.

Algorithm 2 shows a procedure to generate a feasible solution at each step of the process. The main idea is to open nodes and edges while it remains some available budget for the period. The way to open nodes is given by sorting the non-open ones in non-increasing order with respect to the flow originated at each node. Connections are built between the closest already open nodes.

Although there is no explicit mention in Algorithm 2 to the continuous variables $\{z^r\}$ and $\{p^r\}$, these variables can be obtained using the decision variables $\mathbf{x}$ and $\mathbf{y}$, as the percentages of budget and population covered at each period. Thus, they are only implicitly used in the heuristic algorithm.

Eventually, in the last period (i.e., when $t=T$) may be necessary to force the opening of some nodes if the corresponding constraint is not fulfilled. Hence, if at this last period, not all the nodes have been opened, the algorithm forces to open them, until this constraint holds.

3.2. Improving phase

The improving phase of the algorithm uses the successive solutions obtained in the construction phase to build a better feasible solution having a value for the objective function closer to the optimal one.

This phase is based on the fact that the cost to build an edge is proportional to the distance between its end-nodes, so we can define as a neighborhood for each node the ball centered at this node with a given radius.

The improving phase consists of randomly choosing between two classes of movements. In the first movement (deg1) we select an edge and the algorithm tests whether changing an end node (station) improves the solution (see Fig. 1(a)). In the second movement (deg2) we select a path with two connected edges and the admissible changes are to replace the path by a single edge or by a different 2-edge path with the same initial and final nodes. Fig. 1 shows the possible changes over the edges of the graph. Fig. 1(a) corresponds to changes deg1, and it means to change one of the end-points of the given edge (i,j) by another node, say ℓ, and so to change the edge. Figs. 1(b) and (c) describe the changes produced by deg2, and either change two adjacent edges by another two, maintaining the same end nodes, or to replace those two edges by only one edge with extreme points the initial and final nodes of the path. The above two classes of movements are combined to perform the search within the given neighborhood. The parameter `type`, that ranges in {1,2,3,4}, defines the exact form of this combination. Choosing `type`=1 means that only movements deg1 are allowed, whereas `type`=2 stands for only movements of the class deg2. `type`=3 and `type`=4, respectively, alternate sequentially movements deg1 and deg2; and deg2 and deg1.

We show the pseudocode for the improving phase in Algorithm 3. We denote by $cost_{(i,j),\ (i,l)}(t)$ the corresponding construction cost of changing edge (i,j) by edge (i,l) at period t in the incumbent solution (taking into account also the cost of opening node l and closing node j). Analogously, $cost_{(i,j,l),(i,s,l)}(t)$ denotes the construction cost of changing edges (i,j) and (j,l) by edges (i,s) and (s,l) in the incumbent solution (taking into account also the cost of opening node s and close node j). This notation extends to the corresponding flow costs as $flow_{(i,j),(i,l)}(s)$ and $flow_{(i,j,l),(i,s,l)}(s)$, respectively. In Algorithm 3, the parameter γ allows local solutions, in any improvement step, to exceed the previous value of the construction cost (in order to obtain a better global solution). γ has been taken equal to 1.15, so that the incumbent value of the construction cost can exceed a 15% the previous cost.

Algorithm 3. `improvement(type)`

Input: `partialsolution`(t)
for *Each edge (i,j) in the incumbent solution* **do**
  if `type` $=1$ **then**
    Choose a non – open node, l, that is allowed to be connected with
    i and such that $cost_{(i,j),(i,l)}(s) \leq \gamma cost(s)$ and
    $flow_{(i,j),(i,l)}(s) \leq flow(s)$. Then, open node l, close node j,
    remove edge (i,j) and add edge (i,l) at period t.
  if `type` $=2$ **then**
    Choose an open node, l, that is connected with i, and
    choose another non – open station, s, that is allowed
    to be connected with i and l (replacing j by s) and such
    that $cost_{(i,j,l),(i,s,l)}(s) \leq \gamma cost(s)$ and $flow_{(i,j,l),(i,s,l)}(s) \leq flow(s)$.
    Then, open node s, close node j, remove edges (i,j) and
    $(j,l)$and add edges (i,s) and (s,l) at period t.
    If there are no non – open nodes with those
    requirements, change,
    if possible, edges (i,j) and (j,l) by (i,l) and close node j.
if `type`$=3$ **then**
  Compute first the movements of `type` $=1$ and
  then, movements of `type` $=2$.
if `type`$=4$ **then**
  Compute first the movements of `type` $=2$ and then,
  movements of `type` $=1$.
Output: Improved solution at period t.

3.3. Computational experiments

The computational tests presented in this section have been designed in order to evaluate the performance of the solution

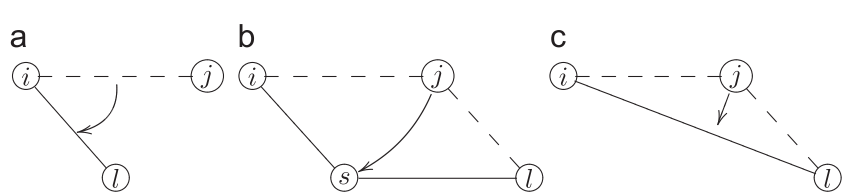

Fig. 1. Changes over the edges of the network at each iteration.

procedure developed in Section 3. To this end, the algorithm was implemented using *Visual C++* 6.0, and all computational tests have been performed on a PC with a *Pentium IV* processor with 2.0 GHz and 2 MB of RAM. We use as optimization solver XPRESS Mosel v2.2.0.

Moreover, we report on a randomly generated computational experiment. In the design of this experiment we identify two relevant factors, namely the total number of nodes and the planning horizons. For each of these factors we consider different levels that define our battery of test problems: M varies in {5, 10, 20, 30, 50, 75, 100} and T in {2, 5, 10, 15, 20, 25, 30} satisfying $M/4<T<3M/4$. For each combination of factors and levels we generate five instances. In total we have generated 80 instances using the following structure based on the real data obtained from the expansion of the Spanish high-speed railway network:

- Set up costs of new stations drawn from a uniform distribution in [10, 50].
- Building costs of 1 km drawn from a uniform distribution in [5, 15].
- Population drawn from a uniform distribution in [50 000, 1 000 000].
- Minimal and maximal distances are computed, respectively, as the tenth and ninetieth percentiles of the sorted list of distances.

Each city is associated with a pair of coordinates drawn from an uniform distribution in [30, 50] $\times[-10, 3]$, and the distances between them are given by the geodisical approximation $d_{ij}=1.6095(69.1^2(x_i-x_j)^2+53.0^2(y_i-y_j)^2)^{1/2}$, for each pair of cities with coordinates (x_i, y_i) and (x_j, y_j). Note that the usage of any other distance would not change the conclusions of our computational tests.

Flows are computed using generalized gravitational models described in Section 4: $w_{ij}=m_i^T(m_j^T)^{1+\gamma}/d_{ij}^{\lambda}$ with $\gamma=0.2$ and $\lambda=1.2$. (The interested reader is referred to [28] for further details.)

The total population is the sum of all the populations. The lower bound for the proportion of population to be covered is pTOT=0.9, the total distance, dTOT is set to dMAX times the number of cities and the total budget to be spent is B = e^tM + dTOT c^t. To build the initial configuration, we select, randomly, 10% of the total number of cities. Then, they are connected attending to the following scheme: Let i be one of the selected cities. Connect i to j being j the nearest city, in the chosen set of cities, that has not been connected yet. Repeat the above process with the city j, until all cities in the initial configuration have been connected.

For better evaluation of our results we have used XPRESS to run some additional experiments. The information we wish to obtain with these experiments is: (1) optimal values of the instances; and (2) CPU times required by XPRESS to obtain optimal solutions. (For these tests a maximum CPU time of 10 000 s was fixed.) On the other hand, for the heuristic algorithm we obtain the same information in order to evaluate the quality of the alternative approach.

Table 1 contains a summary of the average results. The first and second columns show the number of nodes and planning horizons of the instances. The third column shows the average CPU times for each problem running B-&-B in XPRESS and the minimum and maximum CPU times obtained for these instances. The fourth column shows the average CPU times for each problem using the heuristic algorithm as well as the maximum and minimum CPU times. The fifth column presents the percent GAP of the objective value obtained with the heuristic algorithm with respect to the exact value obtained with XPRESS.

The results of Table 1 show that the feasible solutions obtained with our algorithm are good (attending to the gap between them and the best solution found by XPRESS) and they are computed in much smaller CPU times. Thus, allowing to efficiently solve larger instances whenever commercial solvers fail to solve them.

An interesting observation of our computational experience is that we found several instances where XPRESS was unable to find even a feasible solution in 10 000 s while our heuristic algorithm does it very efficiently. (This behavior starts with size instances around M=75, T=40, where our approach solves the problem in approximately 50 s.)

Fig. 2 shows the average CPU time as a function of the number of nodes. From this figure, the reader can observe that the CPU

Table 1
CPU times and percent gaps for different instances.

M	T	**time_ex** (min, max)	**time_heu** (min, max)	**GAP** (%)
5	2	0.02 (0.02, 0.04)	0.00 (0.00, 0.01)	6.23
10	2	0.20 (0.13, 0.45)	0.00 (0.00, 0.01)	3.21
	5	1.94 (0.25, 8.20)	0.01 (0.01, 0.01)	5.64
20	5	5.41 (2.10, 11.89)	0.32 (0.28, 0.34)	1.47
	10	58.53 (13.76, 127.32)	0.82 (0.77, 0.86)	4.05
30	10	16.41 (6.06, 46.03)	9.85 (9.09, 10.58)	1.04
	15	348.77 (13.81, 1325.28)	11.20 (9.25, 13.25)	5.60
	20	430.17 (128.22, 936.18)	9.85 (9.09, 10.58)	3.11
50	10	58.85 (28.57, 105.28)	3.13 (3.01, 3.20)	1.63
	15	113.45 (63.66, 189.32)	5.30 (5.17, 5.49)	1.64
	20	261.30 (137.22, 639.62)	7.69 (7.51, 7.85)	1.63
	30	412.81 (93.37, 1072.07)	22.62 (13.49, 31.66)	3.46
75	10	241.76 (166.51, 338.00)	25.46 (9.09, 59.52)	4.68
	25	4902.76 (1817.21, 7938.45)	45.01 (25.64, 74.49)	5.19
100	25	10 526.70 (6102.60, 15318.70)	66.82 (10.58, 98.65)	2.94
	30	9807.71 (7583.97, 12545.30)	96.02 (69.36, 112.20)	3.23

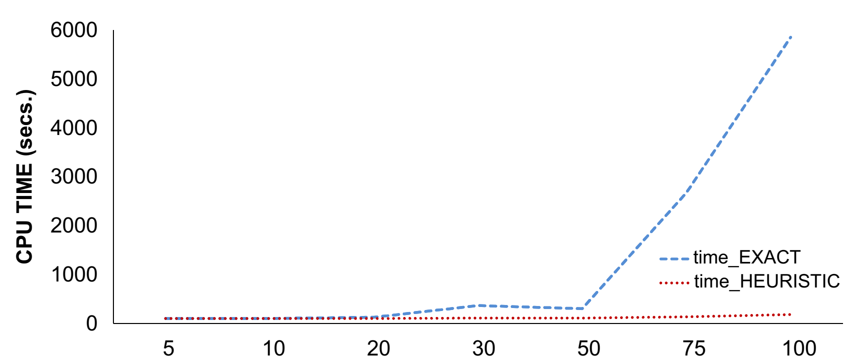

Fig. 2. Average CPU times by number of cities.

times for the exact algorithm increase exponentially with respect to the CPU times of the heuristic algorithm when the number of nodes increases.

4. The Spanish high-speed railway network: a case study

In order to apply our model to the actual case of the Spanish PEIT, in this particular instance $M=47$ and $T=15$ (according to PEIT the planning horizon is 2005–2020 and the "important" cities are the capitals of main-land Spanish provinces). This case study excludes those Spanish provinces located not in the main-land that certainly cannot be connected by train.

The parameters of the general model are set with data obtained from different sources which are specified in the text.

- Cost for 1 km of edge built in each period (CPI is the inter annual Consumer Price Index. According to INE, CPI=0.031 at November 25, 2005): $c^t = 8.49(1+CPI)^t$ m€, $t \in \mathcal{T}$.
  This cost is estimated with data provided by 'Direccion General de Proyectos, Programación y Construcción de Infraestructuras' of ADIF (Spanish Administrator of the Railway Infrastructure). The final estimation is the average costs per kilometer in those links already built between Madrid–Zaragoza–Lleida and Madrid–Toledo. The reader may note that these costs are net value costs since they are corrected by the CPI in the different periods.
- Distance matrix: d_{ij} is approximated using data from ADIF.
- Costs to built a station in a city at each period: $e_i^t = 49.94$ $(1+CPI)^t$ millions €, $t \in \mathcal{T}$.
  This cost is estimated with data provided by ADIF (Spanish Administrator of the Railway Infrastructure). The final estimation results from averaging the costs for newly built and reconstructed stations: Guadalajara, Yebes, Zaragoza Delicias (remodeled), Madrid Puerta de Atocha, Calatayud, Lleida Pirineos and Toledo. The same comment as for the links construction costs also applies here.
- Population of each city at each period: m_i^t. Source [a]: INE Population census of January 1st, 2005.
- Total population: $\mathtt{P} = 41\,016\,355$ inhabitants.
- Total budget: $\mathtt{B} = 83\,450$ m€.
- Upper bound on the maximal edge length: $\mathtt{dMAX}=300$ km.
- Lower bound on the minimal edge length: $\mathtt{dMIN}=30$ km.
- Lower bound on the overall length of the network: $\mathtt{dTOT}=10\,000$ km.
- Lower bound on the overall population covered by the network: $\mathtt{pTOT}=90\%$.
- Estimated flow from cities i to j (using a gravitation model): $w_{ij} = m_i^T (m_j^T)^{1+\gamma}/d_{ij}^{\lambda}$. Where $\lambda \in [1,2]$ and $\gamma \in [0,1]$.[1]
- Overall flow originated at the city i: $O_i = \sum_{j=1}^{n} w_{ij}$.
- Overall flow with final destination at the city j: $D_j = \sum_{i=1}^{n} w_{ij}$.
- Operation costs: $\alpha = \varepsilon = 5 \times 10^{-4}$, $\beta = 5 \times 10^{-5}$.

In our implementation we have used an approximation of the distances between cities based on distances as used by ADIF.

Our model also uses data on origin–destination flows of passengers between cities using the high-speed train connection in the long run, i.e. when the network is completed. Unfortunately, these data were not available and thus, we have estimated them using a gravitational model based on forecasted population. Our estimates are of the form $w_{ij} = m_i^T (m_j^T)^{1+\gamma}/d_{ij}^{\lambda}$, with $\lambda \in [1,2]$ and $\gamma \in [0,1]$. In this case study the parameters are set to $\lambda = 1.2$ and $\gamma = 0.2$ to adequately scale construction and operation costs.

[1] Setting parameters $\lambda = 2$ and $\gamma = 0$ we have the Huff model (see [28] for a detailed analysis).

Setting these values for the parameters, the software XPRESS Mosel v2.2.0. found the final solution proposed by the model. Solutions are represented graphically in Figs. 1–4 where the upper part corresponds to the one obtained with our model whereas the lower part is the one actually implemented by PEIT. We represent the links already available in the base year with continuous lines whereas the new links opened at the current period are depicted with dashed lines. The final solution obtained with such a software appears graphically in Fig. 6. That figure shows the stations and links that the model considers optimal to be opened, under the given conditions.

To a better understanding of our results, for readers not familiar with the Spanish railway network, we first compare the results obtained by our model (see Fig. 6) with three of the most well-known national high speed railway networks, namely those in France, Germany and Japan. In Table 2 we show the frequency distribution of edge lengths as a proxy for the character of the demand in each one of these networks. First of all, we point out that data from the German high speed network are somehow non-comparable since there are few edges of actual high speed in that network (speed faster than 250 km/h). Nevertheless, we have included this information for the sake of completeness. On the other hand, we observe that the results predicted by our model are rather similar to those in the French and Japanese networks, except perhaps in the shortest and longest connections. For the former, we found a 45% of the edges within this range, all of them with lengths larger than 50 km, whereas in France and Japan the percentages are around 20%. This is due to distribution of medium size cities in Spain around major cities, namely Madrid, Barcelona, Bilbao, Valencia and Seville. Moreover, for the latter we did not predict any edge longer than 300 km, mainly due to the requirement of PEIT of having a station in any of the capitals of Spanish provinces. (The reader may note that in mainland Spain there are 52 provinces and the maximum of the minimum distance between two of such cities is smaller than 300 km.)

To compare the expansion of the Spanish high speed network obtained by means of our model with the one adopted by PEIT, we must compare the first stage of our model with the network already finished in 2006 in the actual situation as well as the second and the third stages of our model with the sections actually finished in 2007 and 2008. Finally, we compare the solution of our model with the final network proposed by PEIT for the year 2020. Note that no other comparisons are possible since PEIT does not announce intermediate stages of the network expansion.

4.1. Comparison with the results of PEIT

4.1.1. Comparison status after year 2006

The sections projected by our model for this first period are: Albacete–Madrid, Albacete–Teruel, Cuenca–Madrid, Cuenca–Teruel, Guadalajara–Soria, León–Orense, León–Palencia y Lleida–Tarragona. The above-mentioned situation differs from the one obtained by PEIT for this period. Note that during the year 2006, PEIT suggests to construct only the section Lérida–Tarragona, as we can observe in the comparative Fig. 3.

The proportion of covered population, in our solution, up to the first period is of 13.33%, with a proportion, of 12.93% on the total budget invested at this first stage (i.e. an investment of 9434.76 m€).

4.1.2. Comparison status after year 2007

The sections projected by our model for the second period are: Ávila–Toledo, Huelva–Badajoz, Sevilla–Huelva, León–Zamora, whereas the sections that were built by PEIT in 2007 are Madrid–Segovia–Valladolid, and Córdoba–Málaga, as we can observe in the comparative Fig. 4.

The proportion of covered population, on the whole, up to the second period is of 21.63%, with a proportion of 9.79% on the total

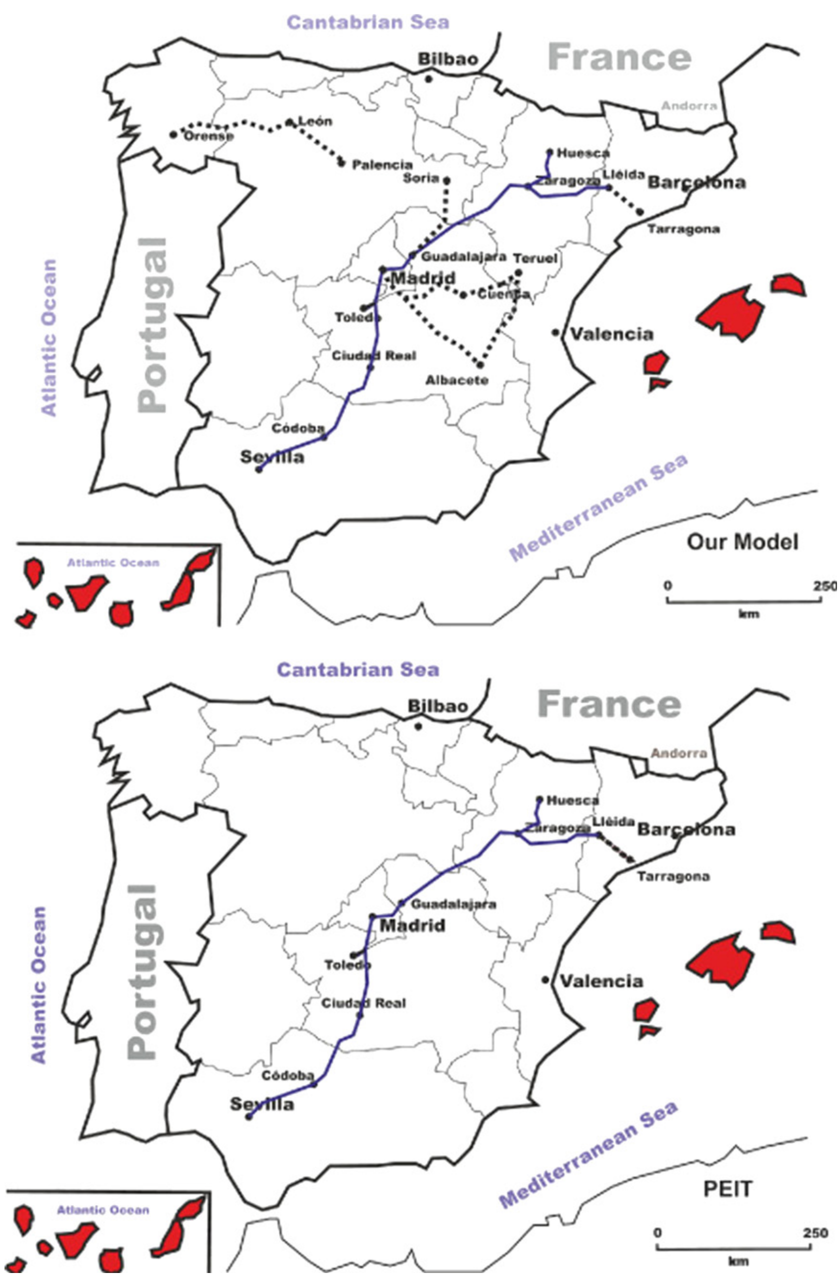

Fig. 3. Comparison after first stage.

budget invested at this second stage (i.e. an investment of 7143.57 m€).

4.1.3. *Comparison status after year 2008*

The sections projected by our model for the third period are: Ávila–Salamanca, Badajoz–Cáceres, Burgos–Palencia, Cáceres–Salamanca, Salamanca–Zamora, whereas the section that has been built by PEIT in 2008 is Tarragona–Barcelona, as we can observe in the comparative Fig. 5.

The proportion of covered population, on the whole, up to the third period is of 25.93%, with a proportion of 3.89% on the total budget, invested at this third stage (i.e. an investment of 2837.64 m€).

4.1.4. *Final stage year 2020*

Since PEIT does not announce (in fact it is not yet fixed) the intermediate stages in the expansion, we can only compare the final picture of both approaches. First, the total cost of our solution is cheaper than the one given by PEIT. In fact, the construction cost of our solution produces a saving of 12.56% on the forecasted cost proposed by the Ministry.

In addition, PEIT requires a coverage of population of at least 90%, fulfilling our network the above-mentioned requirement.

The reader may note that the model proposed for the Spanish high speed network expansion is nothing but a tool for helping in the decision-making process. Obviously, the quality of the outputs provided by this tool depends on the accuracy in the modeling phase of the real-world problem.

4.2. *The heuristic approach*

We have also used the heuristic algorithm described in Section 3 to obtain an approximate solution for the case study of the Spanish railway network. The goal is to compare the solution

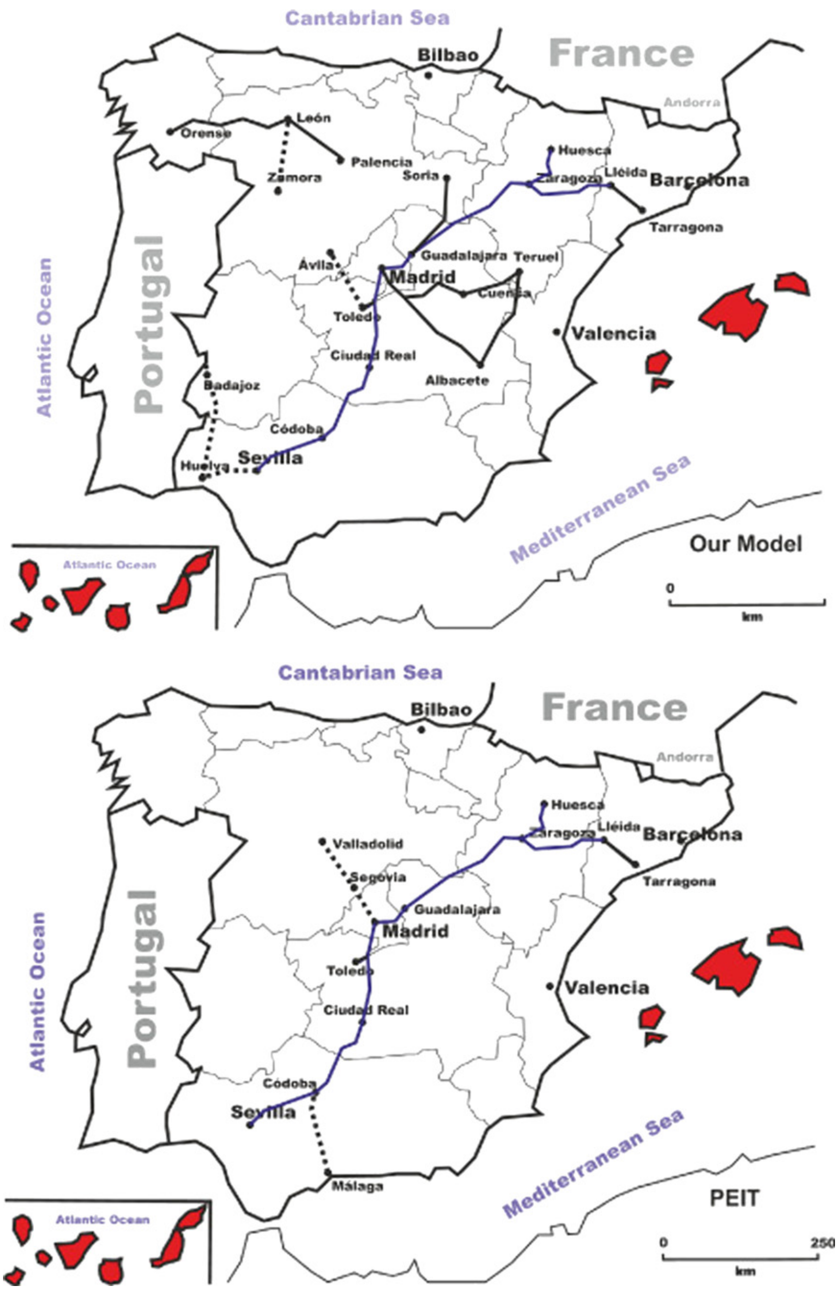

Fig. 4. Comparison after second stage.

Table 2
Comparing high speed networks in France, Germany, Japan and Spain.

Kms	Frequency			
	Spain	**France**	**Germany**	**Japan**
<100	**0.45**	**0.24**	**0.82**	**0.19**
100–200	**0.44**	**0.47**	**0.18**	**0.44**
200–300	**0.11**	**0.18**	**0.00**	**0.19**
>300	**0.00**	**0.12**	**0.00**	**0.19**

proposed by our heuristic (see Fig. 7) with the solution given by the exact model (see Fig. 6). We observe that the solution produced by the heuristic (Fig. 7) is rather similar to the one obtained from the exact model. This is explained by the fact that the heuristic algorithm is designed to capture the essence of the model and thus, to provide good approximate solutions for it. In Table 3 we compare, for each period, the costs of opening stations and edges given by the heuristic algorithm and the exact approach. The overall cost with the former, 74 793.64 m€, is slightly larger than the one obtained by the latter, 72 968.68 m€. This difference means a percent GAP of 2.43%. Comparing the intermediate solutions at each stage, we note that except in two periods, the heuristic algorithm obtains a cost of opening stations at each stage larger than those obtained with the exact implementation. However, for the cost of building edges, we got the opposite situation. In both cases (edges and stations), the largest difference is with respect to the last period, where the heuristic needs some updating phase (opening some extra stations and/or edges) to make the final solution feasible, i.e. fulfilling the required coverage and final number of open stations.

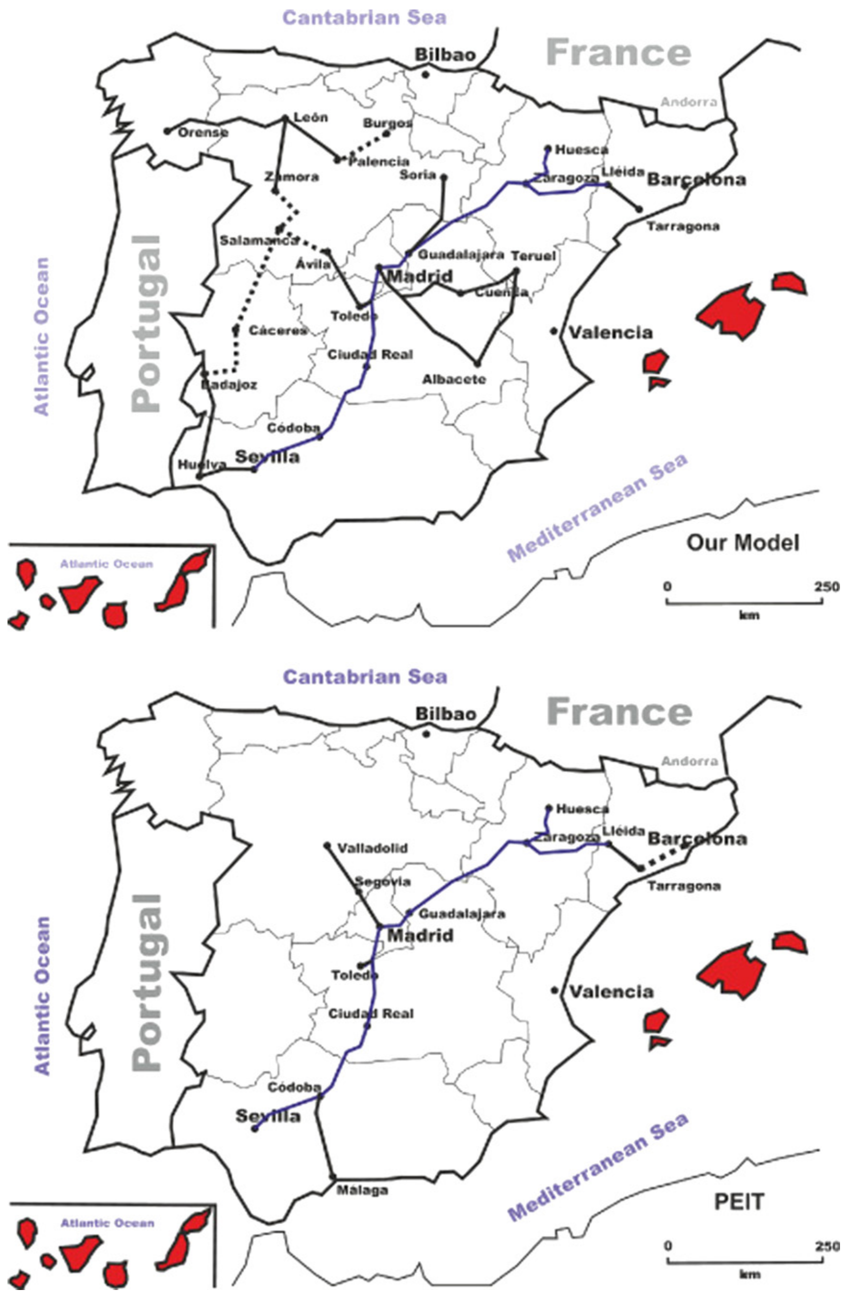

Fig. 5. Comparison after third stage.

However, even with this last extra cost, the overall GAP after the 15 periods is rather small (less than 3%).

We point out that in both solutions there are stages where no stations are opened: periods 5, 6 and 12 for the exact algorithm and periods 7, 12 and 13 for the heuristic method. In addition, the heuristic approach does not open any edge at period 14. The reader may note that this situation may be, in some cases, advisable whenever it is better to wait for a future stage to open an edge or station than open them at present time because they do not help in reaching the goals at the current stage and suppose an additional cost.

5. Conclusions

PEIT project has several important objectives which are distributed political issues, social issues, sustainability and economic development and competitiveness. These are very important goals for a country and they make the problem very complex. Nevertheless, most of these objectives are not publicly available or they are simply stated as imprecise political promises. The above makes it difficult to compare the "quality" of the solutions provided by the two approaches (PEIT and ours). Our approach is entirely quantitative and it only takes into account publicly available guidelines published by the Spanish Ministry of Public Works. For this reason, we cannot simply claim that our solution is better than the one announced by PEIT. In this regards, we must be cautious on the conclusions drawn from the comparisons. This point raises a recommendation for future expansion plans in other regions: To state in a precise way all goals of all types (social, environmental, economic…) pursued by these policies so that they can be incorporated in quantitative analysis. In any case, our approach, with its current limitations, seems to be, at least, a good tool to compare different scenarios as

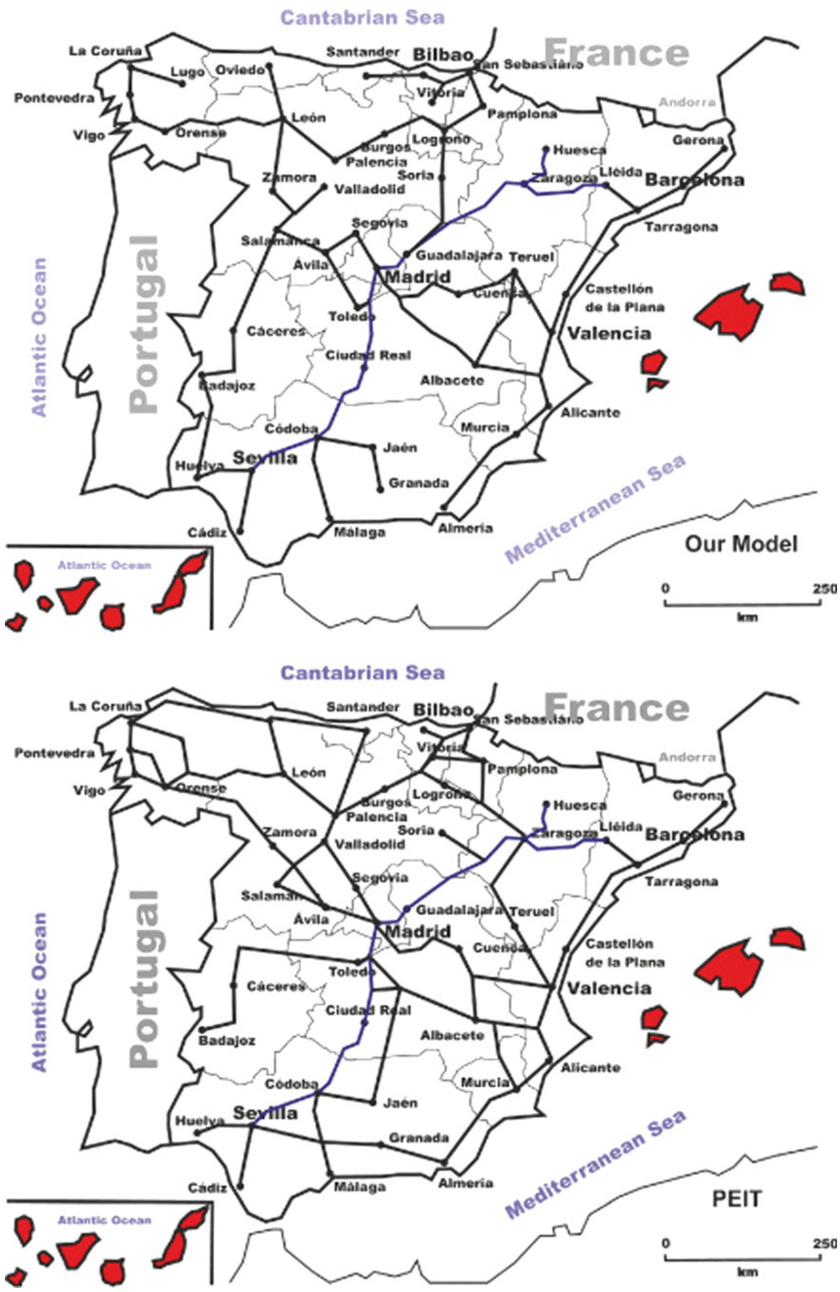

Fig. 6. Final comparison.

a what-if decision support tool in the decision making process of this network expansion.

In the following we draw some conclusions about the intermediate and final stages in the solution of our model applied to the Spanish high-speed railway network case study.

Our approach gives an alternative solution for planning the expansion of the Spanish high-speed railway network, fulfilling the requirements stated in PEIT. This approach may be taken as an informative supporting tool for the decision-makers (among many others). The reader may note that this type of decision support systems cannot be taken as a fully normative approach since in its design one may neglect some factors that do not appear, explicitly, in the official Strategic Planning of the Ministry but that may affect the decision-making process.

First of all, we observe that our intermediate solutions need not to be connected, see e.g. Fig. 4 (the model does not require this property in intermediate stages) although connectivity is ensured in the last stage (end of the planning horizon). This is a first difference with the solution actually implemented by PEIT where, although not explicitly announced, the partial expansions are always connected networks.

One can appreciate some differences between our solutions and the one approved by the Ministry. Concerning the topology both networks are somehow similar in that they show a radial structure. However, there are differences in the connectivity pattern. In particular, in PEIT, we observe that there exists a *corridor* connecting cities in the Cantabrian coast whereas it does not exist a direct connection along the western north–south edge (parallel to the Portuguese border). Other than that both proposals are rather close. The fact that our solution comes out of a pure quantitative model where we have only imposed requirements publicly available and no other further constraints (of *political* type) makes the similarity of both solutions rather interesting. On the other hand, it seems rather likely that some

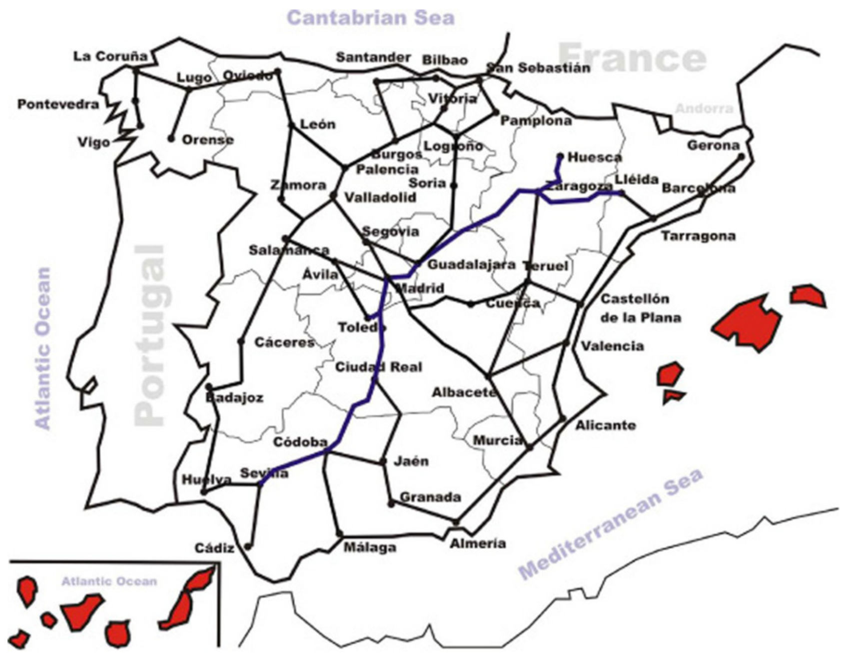

Fig. 7. Solution for the Spanish network using the heuristic approach.

Table 3
Cost of opening stations and edges by using the exact and the heuristic approaches for the Spanish high-speed railway network.

Period	Exact		Heuristic	
	Cost of stations	Cost of edges	Cost of stations	Cost of edges
1	588	8846.76	1190.51	7854.24
2	151.41	6992.16	766.39	4596.95
3	103.96	2733.67	631.50	2674.50
4	267.71	4017.65	487.84	4081.09
5	0.00	4429.07	167.49	2679.74
6	0.00	4516.42	517.55	4134.80
7	234.03	4126.35	0.00	2999.02
8	60.26	4410.19	732.09	5293.78
9	62.07	4377.12	188.51	4080.58
10	255.73	4059.80	582.50	3630.41
11	131.70	4657.54	199.99	2635.09
12	0.00	4787.71	0.00	2494.43
13	69.86	4312.94	0.00	7681.17
14	143.91	4234.66	218.53	0.00
15	74.11	4323.79	900.38	13 374.45

decisions in the actual implementation of PEIT are based on political (not necessary objective or quantitative) reasons. This would explain some of the differences, as for instance not having the western north–south edge.

Concerning population coverage and budget investment through the planning horizon we observe a different pattern in both solutions. In our proposal, a large amount of the population and budget are covered during the initial periods. This is due to the increment of prices by the CPI (the inter annual Consumer Price Index). The model finds that for minimizing the overall cost the optimal budget investment policy is "the sooner the better". This reason is also valid for population since population and budget are clearly related and any decision on budget affects the population coverage and therefore also the passenger flows between cities.

As mentioned above, the total population coverage after the entire expansion (year 2020) in our solution is above 90%, as required in PEIT. Moreover, with our solution, 12.56% of the construction cost is saved over the total budget of the Ministry.

Once again, we would like to insist that it is difficult to compare the "quality" of both solutions since our approach is entirely quantitative and it only takes into account publicly available guidelines published by the Spanish Ministry of Public Works. For this reason, we cannot simply claim that our solution is "better/worse" than the one announced by PEIT. Nevertheless, according to our estimated data, it seems to be cheaper and still fulfilling all the requirements. In this regards, ours is better but we could have missed some political or simply non-written constraints that may make our solution actually not feasible. In any case, our approach seems to be a good tool to compare different scenarios as a decision support tool in the decision making process of the network expansion.

Finally, we would like to mention that in the strategic planning of such a type of network it would have being advisable to consider social welfare and environmental issues at the same level that construction and operation costs. In our case study this resulted very hard to implement due to lack of data. Nevertheless, this point has become an interesting challenge that will be the topic for future research.

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[b] Spanish Statistics Institute. INEbase. Consumer Price Index, CPI, variation from November, 2004 until November, 2005. http://www.ine.es/daco/ipc.htm.

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Acknowledgements

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